

LUMINOSITY LOSS RATE
DUE
TO INTRABEAM SCATTERING

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due
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In the limit of short interaction region the luminosity can be very well represented by

$$L = \frac{N^2 B f_{rev}}{4\pi \sigma_v^* \sigma_H^*}$$

where, for identical colliding beams,

- N , number of particles per bunch
- B , number of bunches per beam
- f_{rev} , revolution frequency
- σ_v^* , vertical rms beam size at the collision point
- σ_H^* , horizontal rms beam size at the collision point

assuming crossing ~~at~~ at an angle on the horizontal plane

$$f = \sqrt{1 + p^2}$$

and

$$p = \frac{\alpha \sigma_e}{2 \sigma_H^*}$$

(2)

α , total crossing angle
 σ_e , rms bunch length.

Intrabeam scattering has the effect of having σ_v^* , σ_H^* and σ_E , the r.m.s. energy spread to vary, generally by growing.

~~Obviously~~ Define the luminosity loss rate

$$\tau_L^{-1} = -\frac{1}{L} \frac{dL}{dt}$$

Obviously:

$$\tau_L^{-1} = \frac{1}{\sigma_v^*} \frac{d\sigma_v^*}{dt} + \frac{1}{\sigma_H^*} \frac{d\sigma_H^*}{dt} + \frac{1}{f} \frac{df}{dt}$$

One can work out that

$$\frac{1}{f} \frac{df}{dt} = \frac{P^2}{f^2} \left[\frac{1}{\sigma_e} \frac{d\sigma_e}{dt} - \frac{1}{\sigma_H^*} \frac{d\sigma_H^*}{dt} \right]$$

Assumptions:

1. The bunch length increases (or decreases) at the same rate the energy spread does, that is

$$\frac{1}{\sigma_e} \frac{d\sigma_e}{dt} = \tau_e^{-1}$$

2. We assume that the two modes of oscillations (H and V) are fully coupled to each other on a time shorter than the intrabeam scattering diffusion ~~is~~ times, that is

$$\frac{1}{\sigma_v^\alpha} \frac{d\sigma_v^\alpha}{dt} = \frac{1}{\sigma_H^\alpha} \frac{d\sigma_H^\alpha}{dt} = \tau_H^{-1} + \tau_V^{-1}$$

Here τ_e^{-1} , τ_H^{-1} and τ_V^{-1} are the diffusion (or damping rates) from intrabeam scattering respectively in energy, horizontal (betatron) size and vertical (betatron) size.

In conclusion

$$\tau_L^{-1} = \frac{p^2}{f^2} \tau_E^{-1} + \left(2 - \frac{p^2}{f^2}\right) (\tau_H^{-1} + \tau_V^{-1})$$

For head-on collision $p=0$, $f=1$ and

$$\tau_L^{-1} = 2 (\tau_H^{-1} + \tau_V^{-1})$$